Summerschool on Analysis and Applied Mathematics

12. September - 16. September 2022, in Münster, Germany

Schedule

Lecturer

- **Prof. Dr. Antonin Chambolle:** Rigidity results in linearized elasticity with cracks
- **Prof. Dr. Maria Giovanna Mora:** Nonlocal interaction problems in dislocation theory
- **Prof. Dr. Felix Otto:** Minimizing movement schemes, with an application to the thresholding scheme for mean curvature flow
- **Prof. Dr. Angkana Rüland:** Rigidity, Flexibility and Scaling in the Microstructures in Shape-Memory Materials

Schedule

	Monday		Tuesday		Wednesday		Thursday		Friday
10:30	Registration	9:00	Otto	9:00	Otto	9:00	Rüland	9:00	Otto
11:00	Rüland	10:30	Coffee Break	10:30	Coffee Break	10:30	Coffee Break	10:30	Coffee Break
12:30	Lunch	11:00	Chambolle	11:00	Chambolle	11:00	Otto	11:00	Chambolle
14:00	Mora	12:30	Lunch	12:30	Lunch	12:30	Lunch	12:30	Lunch
15:30	Coffee break	14:00	Mora	14:00	Mora	14:00	Chambolle		
16:00	Rüland	15:30	Coffee & Poster	15:30	Coffee & Poster	15:30	Coffee break		
		16:45	Rüland			16:00	Mora		
		18:30	Reception Dinner			19:00	Social Dinner		

Abstracts

Tuesday 15:30 – 16:45

Bohdan Bulanyi

Limiting behavior of minimizing p-harmonic maps in $3d$ as $p \to 2-$

Abstract: We study the limiting behavior of minimizing p-harmonic maps from a $3d$ domain to a compact Riemannian manifold as $p \to 2-$. In collaboration with Jean Van Schaftingen and Benoît Van Vaerenbergh, we prove that there exists a closed set S of finite length, such that minimizers converge to a locally harmonic map away from S. Furthermore, S restricted to the interior of the domain is a locally finite union of straight line segments.

Andrea Chiesa

Finite-strain Poynting-Thomson model: existence and linearisation

Abstract: We investigate the large-strain Poynting-Thomson model, a viscoelastic system where the total deformation is defined as the composition of a compatible viscous deformation and an elastic one, in accordance with the classical multiplicative decomposition of the total strain. We address the existence theory, calling for the treatment of both Lagrangian and Eulerian variables. Moreover, we perform the linearisation of the model. A remarkable trait of the result is that it does not require non physical second-order gradients. This is joint work with M. Kružík and U. Stefanelli.

Víctor Navarro Fernández

Advection-diffusion equations: optimal stability estimates and convergence of the upwind scheme

Abstract: We develop stability estimates for the advection-diffusion equation in the DiPerna-Lions setting, i.e. with a velocity field that is Sobolev regular in the spatial variable. These estimates are formulated with the help of Kantorovich-Rubinstein distances from the theory of optimal transport with logarithmic cost functions, and provides thus a bound on the rate of weak convergence. First, we obtain an estimate for two distributional solutions with different initial data, vector fields and diffusivity constants. Second, we study the implicit upwind finite volume scheme and we prove an error estimate for the rate of convergence of approximate solutions towards the unique exact solution. Both estimates are optimal in various respects.

Guy Fabrice Foghem Gounoue

Stability of nonlocal nonlinear Dirichlet and Neumann problems driven by integrodifferential p-Lévy operators.

Abstract: We will study the well-posedness of nonlocal nonlinear Dirichlet and Neumann complement value problems on bounded domains governed by symmetric integrodifferential p-Lévy operators. A prototypical example of integrodifferential p-Lévy operators is the well-known fractional p-Laplace operator. Asymptotically, we show that the local nonlinear Dirichlet and Neumann boundary value problems associated with p-Laplacian are L^p-limits of the nonlocal ones. We reach this conclusion by establishing important results like robust Poincaré type inequalities and Gamma convergence of the nonlocal nonlinear energies forms involved to the local ones.

Myrto Galanopoulou

The equations of quasiconvex adiabatic thermoelasticity

Abstract: We study the equations of adiabatic thermoelasticity endowed with an internal energy satisfying an appropriate quasiconvexity assumption, associated with the symmetrisability condition for the system. We prove and use a Garding-type inequality to establish a weak-strong uniqueness result for a class of dissipative measure-valued solutions.

Dimitris Gazoulis

On the Γ-convergence of the Allen-Cahn functional with boundary conditions

Abstract: We study minimizers of the Allen-Cahn system. We begin with the ε-energy functional and under the appropriate Dirichlet conditions that we impose, we construct a minimizer of the limiting problem that possesses three phases via a Γ-convergence result. The Γ-convergence analysis together with some particular estimates allows us to obtain information about the asymptotic behavior of the minimizers.

Max Griehl

Derivation of a Bending Theory for Nematic Liquid-Crystal Elastomeric Plates through Gamma-Convergence

Abstract: Liquid-crystal elastomers (LCEs) are materials whose shape can be controlled via external stimulation thanks to liquid crystal structures inside of polymer chain networks. To describe the deformation and microscopic liquid-crystal behavior of these materials, we start from a three-dimensional model taking into account coupled terms of hyper-elasticity and Oseen-Frank-type energy. Using Gamma-convergence, we then derive a dimension-reduced model, effectively describing thin LCE-plates in terms of a deformation and a liquid-crystal director field. We also take into account different types of boundary conditions for the deformation and the director field during Gamma-convergence

Jonas Ingmanns

The Dean-Kawasaki equation: Numerically capturing density fluctuations in systems of interacting diffusing particles

Abstract: We look at systems with multiple species of particles on the flat torus $[-\pi, \pi)^d$. Each particle is driven by an independent Brownian motion; the particles interact according to smooth interaction potentials. More specifically, we are interested in the random fluctuations of the densities of the particles around the mean field limit (i.e. the best deterministic approximation of the density for large particle numbers).

The Dean-Kawasaki equation a strongly singular SPDE has been proposed in the physics literature to describe these fluctuations. However, even in the simple non-interacting case, the only admissible martingale solutions are the trivial empirical particle densities the Dean-Kawasaki equation does not seem to be anything but an unnecessarily complicated mathematical rewriting of the particle system.

Interestingly, the Dean-Kawasaki equation can be used as a recipe for accurate and efficient numerical simulations of interacting particle systems. More specifically, we prove that a structure-preserving finite difference scheme yields a discrete approximation of the density that captures the fluctuations up to arbitrary order in $N-1$ (in suitable weak metrics; N is the number of particles per species).

Joint work with Federico Cornalba, Julian Fischer, Claudia Raithel.

Lukas Koch

A p-harmonic type approximation to optimal transport with p-type cost

Abstract: I will present a generalization of the harmonic approximation result to optimal transference plans to optimal transport with quadratic cost of Otto and Goldman to costs of with p-growth and p-coercivity.

Lennart Machill

One-dimensional viscoelastic von Kármán theories

Joint Work with Manuel Friedrich

Abstract: We consider a model for nonlinear viscoelastic materials in Kelvin's-Voigt's rheology obeying the system of equations

$$-\text{div}\left(\partial_F W(\nabla w) - \zeta_h \mathcal{L}_p(\nabla^2 w) + \partial_{\dot{F}} R(\nabla w, \partial_t \nabla w)\right) = f \text{ in } [0, T] \times \Omega_h$$

Here, $[0, T]$ is a process time interval and $\Omega_h = (1/2, 1/2) \times (h/2, h/2) \times (\delta_h/2, \delta_h/2)$ is a smooth, thin, bounded domain representing the reference configuration. $w\colon [0, T] \times \Omega_h \to \mathbb{R}^3$ denotes a deformation mapping with deformation gradient ∇w. The stress tensors $\partial_F W$, $\mathcal{L}_p$ and $\partial_{\dot{F}}$ satisfy usual assumptions in nonlinear viscoelasticity, and, in particular, comply with frame-indifference principles. We investigate the model by means of gradient flows in metric spaces, and perform a dimension reduction as $h \to 0$ using evolutionary Γ-convergence. The corresponding result is part of the analysis in [1].

References

[1] M. Friedrich, L. Machill. *One-dimensional viscoelastic von Kárám theories derived from nonlinear thin-walled beams.*
Preprint at https://arxiv.org/abs/2204.10032 (2022)

Wednesday 15:30 – open end

David Meyer
Zero-diffusivity limit for advection-diffusion equations

Abstract: We consider advection diffusion equations with a Sobolev vector field. We show sharp convergence rates for the solutions in the zero-diffusivity limit. The main tool in our proof is a new regularity result for the solutions in terms of logarithmic Besov spaces, which also recovers the optimal regularity in the non-diffusive setting

Raphaël Prunier
Regularity in shape optimization under convexity constraint
Keywords: Shape optimization, isoperimetric problem, convexity.

Abstract: The poster fits in the shape optimization field, which consists in the study of optimization problems where the functional to minimize $F(\Omega)$ takes a subset $\Omega \subset \mathbb{R}^d$ as an argument. The prototype of such problem is the isoperimetric problem, which states that for any fixed volume the perimeter $P(\Omega)$ is minimized by a ball of appropriate volume. We are concerned with proving regularity of the minimizers of a class of isoperimetric problems under convexity constraint. More precisely, we prove $C^{1,1}$-regularity of the minimizers of the sum of the perimeter $P(\Omega)$ and a perturbative term $R(\Omega)$ under convexity constraint, meaning that the form Ω considered in the optimization runs in the class of convex sets. A typical example of such $R(\Omega)$ is given by $-\lambda_1(\Omega)$, with $-\lambda_1(\Omega)$ being the first eigenvalue of the Dirichlet Laplacian over Ω. Regularity of minimizers in this case opens up the way to proving that a ball B is a minimizer, thus proving some kind of stability of the ball for this problem.

Felix Schneppe
The Effect of Adjoint Mismatches in the Chambolle-Pock Method

Abstract: The primal-dual method of Chambolle and Pock is a widely used algorithm to solve various optimization problems written as convex-concave saddle point problems. Each update step involves the application of both the forward linear operator and its adjoint. However, in practical applications like computerized tomography, it is often computationally favourable to replace the adjoint operator by a computationally more efficient approximation. This leads to an adjoint mismatch in the algorithm. We study its effect on the convergence of the method.

Sandeep Kumar Soni
Abstract Friedrichs Operators

Abstract: The theory of abstract Friedrichs operators, introduced by Ern, Guermond and Caplain (2007), proved to be a successful setting for studying positive symmetric systems of first order partial differential equations (Friedrichs, 1958), nowadays better known as Friedrichs systems. Recently, Antonić, Michelangeli and Erceg (2017) presented a purely operator-theoretic description of abstract Friedrichs operators, allowing for application of the universal operator extension theory (Grubb, 1968). In this presentation we shall see a nice decomposition of the graph space (maximal domain) as a direct sum of the minimal domain and the kernels of corresponding adjoints, which might be a significant ingredient in further studies of abstract Friedrichs operators. As an immediate application of the decomposition we shall see a concrete pair of mutually adjoint bijective realisation relative to the given joint pair of abstract Friedrichs operators. We shall also see a complete classification of admissible boundary conditions of classical Friedrichs differential operators in one dimensional scalar case with variable coefficients. This is a joint work with Marko Erceg funded by Croatian Science Foundation.

Chakir Tajani

Optimization approach based on metaheuristic methods for a Robin coefficient problem

Abstract: This work presents numerical optimization algorithm based on metaheuristic methods to solve an inverse problem to reconstruct the Robin coefficient in boundary value problem. It consists of identifying the Robin coefficient on the inaccessible part of the boundary representing the corrosion damage of some specimen material. This problem is known to be severely ill-posed in Hadamard sense, since the existence or uniqueness or the continuous dependence on the data of their solutions may not be ensured. Therefore, the inverse problem is formulated as an optimization one adding a regularization term to the functional, then, several metaheuristic optimization methods including the genetic algorithm, particle swarm optimization are explored to solve the considered problem. The efficiency and accuracy of the proposed methods are assessed by their ability of finding and reconstructing of the results. The effects of some metaheuristic parameters on the optimum solutions are examined.

Francesco Mattesini

On the non existence of a stationary Poisson matching in 2d

Abstract: The optimal matching problem is a classical random variational problem at the intersection of analysis, probability and combinatorics that received interest in the last 30 years. We show that there exists no cyclically monotone invariant (quadratic) matching of two independent Poisson processes in the critical dimension $d = 2$. Our argument relies on a recent harmonic approximation theorem together with the two-dimensional local asymptotics for the bipartite matching problem, for which we provide a new self-contained proof based on martingale arguments.

Kai Richter

Linearization and homogenization of nonlinear elasticity close to stress-free joints

Abstract: We derived asymptotic models w.r.t. linearization and homogenization for hyperelastic materials subject to prestrains, which are nearly resolvable, i.e. close to a so called stress-free joint. The derived linear model features a homogenized perturbation term, which reveals first order information about the minimizers of the energy as well as the homogenized minimizers.

Camillo Tissot

On scaling properties for the two-state problem and for a singularly perturbed T_3 structure

Abstract: We study quantitative rigidity properties of the compatible and incompatible two-state problems for suitable classes of $\mathcal{A}$-free operators. In particular, in the compatible setting we prove that all first order, constant-rank, linear operators with affine boundary data which enforce oscillations yield the typical $\epsilon^{\frac{2}{3}}$-lower scaling bounds. Moreover, building on [RT22, GN04, PP04], we discuss the scaling behaviour of a T_3 structure for the divergence operator.

Bohan Zhou

Efficient and Exact Multimarginal Optimal Transport with Pairwise Costs

Abstract: Optimal transport has profound and wide applications since its introduction in 1781 by Monge. Thanks to the Benamou-Brenier formulation, it provides a meaningful functional in the image science like image and shape registrations. However, exact computation through LP or PDE is in general not practical in large scale, while the popular entropy-regularized method introduces additional diffusion noise, deteriorating shapes and boundaries. Until the recent work [Jacobs and Leger, A Fast Approach to Optimal Transport: the back-and-forth method, Numerische Mathematik, 2020], solving OT in a both accurate and fast fashion finally becomes possible. Multi-marginal optimal transport is a natural extension from OT but has its own interest, and is in general more computationally expensive. The entropy method suffers from both diffusion noise and high dimensional computational issues. In this work with Matthew Parno, we extend from two marginals to multiple marginals, on a wide class of cost functions in the form of summed pairwise costs. This new method is fast and does not introduce diffusion. As a result, the new proposed method can be used in many fields those require accurate approach to MMOT.

Dominik Winkler

Invariant Manifolds for the Thin Film Equation

Abstract: We examine the long-time behavior of solutions to the thin film equation with linear mobility in the complete wetting regime on the whole space $\mathbb{R}^N$. In fact, we determine the speed of convergence towards the self-similar solutions. To identify sharp, uniform convergence rates and higher-order asymptotics, we construct invariant manifolds around the self-similar solutions, which meet the eigenspaces of the linearized equation tangentially at the origin.