Adjoint-Based Enforcement of State Constraints in PDE Optimization Problems

Abstract:

Adjoint-based methods have become a workhorse in the solution of unconstrained PDE optimization problems. They make it possible to conveniently determine the gradient (sensitivity) of the objective functional with respect to a control variable, which can then be used in various gradient descent algorithms. Unlike most constraints imposed on the control variable, constraints on the state variables are generally harder to satisfy since they define, via solutions of the governing system, complicated manifolds in the space of control variables. In this talk, we will demonstrate how this traditional adjoint-based framework can be extended to handle general constraints on the state variables. This is accomplished by constructing a projection of the gradient of the objective functional onto a subspace tangent to the manifold defined by the constraint. We focus on the "optimize-then-discretize" paradigm in the infinite-dimensional setting where the required regularity of both the gradient and of the projection is ensured. This proposed approach will be illustrated first with a simple test problem describing optimization of heat transfer in one direction and then a more involved problem where an optimal closure is found for a turbulent flow described by the Navier-Stokes system in two dimensions.

Joint work with Bartosz Protas.