Workshop “Ideal Structure of C*-algebras from Dynamics and Groups” April 8 - 12, 2024

Abstracts for Mini-courses

Yair Hartman (Ben-Gurion University)

Furstenberg's Boundary Theories: Buy one, get one free

Abstract: During the 60s and the 70s, Furstenberg developed two parallel theories regarding a group's boundaries of different flavours. One is topological, known as the Furstenberg boundary, and the other is measurable and relates to random walks, called the Furstenberg-Poisson boundary. Both have universal properties (injectivity \Zimmer amenability), and so, their structure reveals various groups' properties, and their study relates to rigidity. The research of these two theories and their connections with Operator Algebra theory is still very active, yet many questions are open.
To better understand the connections between these two, I'll show that they share the same driving force. We will develop one machinery to produce them both simultaneously, despite the differences between them. Two Boundary Theories for the price of one.

Mehrdad Kalantar (University of Houston)

Stationary C*-dynamical systems and noncommutative boundary maps

Abstract: We approach the problem of C*-simplicity from the following general and simple categorical point of view: if there is a unique morphism \psi from an object A to an injective object C, then \psi factors through any other object B for which there is a morphism from A to B. That implies the “kernel” of \psi is the “unique largest” among all kernel of morphisms going out of A.

We apply this idea to two types of morphisms in the category of G-C*-algebras for a given discrete group G: 1. equivariant ucp maps, and 2. \mu-stationary states for a probability measure \mu on G. The first approach is related to the notion of Furstenberg boundary, and the second is related to the Poisson boundary.

In these series of lectures, we intend to explain, in some details, the main ideas behind these approaches, and their most significant applications in problems of determining ideal and trace structures of C*-algebras generated by unitary representations.

Matthew Kennedy (University of Waterloo)

Boundary Theory for C*-Algebras

Abstract: The characterization of C*-simple groups revealed a deep connection between the structure of reduced C*-algebras of discrete groups and Furstenberg's theory of topological boundaries. This subsequently inspired the the development of a very general theory of boundaries for C*-algebras that, in recent years, has led to a significantly improved understanding of the structure of crossed product C*-algebras and groupoid C*-algebras. In this lecture series, I will provide an overview of these developments, including some of the important underlying technical details and open problems.

Bartosz Kwaśniewski (University in Bialystok)

Aperiodicity, topological freeness and ideal structure for C*-inclusions

Abstract: The mini-course will be mostly based on results from my papers with Ralf Meyer, hopefully providing also a perspective for further research. The topics I plan to cover include:

- General relationship between ideals, as well as primitive ideals, in two C^*-algebras forming a C^*-inclusion $A\subseteq B$; the quasi-orbit space and existence of a quasi-orbit map for regular inclusions.
- Relationships between topological freeness, aperiodicity (proper outerness) and the intersection property for discrete group actions on C^*-algebras, and their generalizations to inverse semigroup actions by Hilbert bimodules and to general C^*-inclusions.
- Relationships between aperiodicity, the almost extension property and uniqueness of pseudo-expectations for general C^*-inclusions.
- Applications to pure infiniteness, noncommutative Cartan inclusions and C^*-irreducible inclusions.

Jiawen Zhang (Fudan University)

Ghostly ideals in groupoid C*-algebras

Abstract: In this mini course, we focus on a class of ideals (called ghostly ideals) in the reduced groupoid C*-algebra C*_r(G) for a locally compact Hausdorff and \'{e}tale groupoid G, motivated by the case of coarse groupoid. We use these ideals to provide a sandwiching result for ideals in C*_r(G), and to study associated open subsets of the unit space. We also apply our results to several families of ideals including tracial ideals and regular ideals. Finally, we would like to calculate some concrete examples if time permits. This is based on a joint work with Kang Li.

Abstracts for invited Talks

Ryoya Arimoto (Kyoto University)

Simplicity of crossed products of the actions of totally disconnected locally compact groups on their boundaries

Abstract: Results of Archbold and Spielberg, and Kalantar and Kennedy assert that a discrete group admits a topologically free boundary if and only if the reduced crossed product of continuous functions on its boundary by the group is simple. In this talk, I will show a similar result for totally disconnected locally compact groups.

Kevin Aguyar Brix (Lund University)

Maximal ideals in reduced group C*-algebras

Abstract: I will discuss recent work with Chris Bruce, Kang Li, and Eduardo Scarparo in which we study the maximal ideals of the reduced C*-algebra of discrete groups via the Furstenberg boundary of the group. As a consequence, we gain some insight into the ideal structure of the redcued C*-algebra of Thompson's group T.

Chris Bruce (University of Glasgow)

Connes’ adele class space remembers the number field

Abstract: The multiplicative group of a number field acts by multiplication on the full adele ring of the field, and the quotient space for this action is Connes’ adele class space. I will present on joint work with Takuya Takeishi in which we prove that the crossed product C*-algebra associated with the adele class space completely remembers the number field in the sense that two such crossed products are isomorphic if and only if the underlying number fields are isomorphic. Primitive ideals and subquotients play a central role in our proof.

Johannes Christensen (KU Leuven)

Ideals in groupoid C*-algebras and their isotropy fibres

Abstract: For any ideal in the C∗ -algebra of a locally compact étale groupoid one can associate an open invariant subset of the unit space of the groupoid. This associated open invariant set plays a key role in most attempts to describe the ideal structure of the C∗ -algebras of locally compact étale groupoids. In a similar fashion to how one associates a subset of the unit space to an ideal, it turns out one can also associate certain isotropy bres. In this talk I will report on a joint project with Sergey Neshveyev where we investigate how any ideal in a locally compact étale groupoid C∗ -algebra naturally denes a family of ideals in certain group C∗ -algebras of isotropy groups. We call this family of ideals the isotropy bres of the ideal of the groupoid C∗ -algebra. I will reect on to which extent these isotropy bres determine the ideal and I will illustrate how this question is connected to the study of certain norms on the group algebras of the isotropy groups of the groupoid. As an application I will parametrize the maximal ideals of any locally compact étale groupoid C∗ -algebra and describe all primitive ideals for a class of graded groupoids.

Anna Duwenig (KU Leuven)

Cartan subalgebras for non-effective twisted groupoid C*-algebras

Abstract: The reduced C*-algebra of an effective twisted groupoid has a canonical Cartan subalgebra: functions on its unit space. A remarkable Weyl groupoid construction, due to Kumjian and Renault, asserts the converse: If a C*-algebra A admits a Cartan subalgebra, there exists such a groupoid whose C*-algebra is isomorphic to A in a Cartan-preserving way. In this talk, I discuss how subgroupoids of (not necessarily effective) groupoids can give rise to Cartan subalgebras. If time permits, I will further give a description of the associated Weyl groupoid and twist in the case of 2-cocycles. Based on joint works with Gillaspy, Norton, Reznikoff, Williams, Wright, Zimmerman.

Francesco Fidaleo (Università di Tor Vergata, Roma)

Spectral actions for free particles and their asymptotics

Abstract: For the spectral action consisting of the average number associated to the gas of free q-particles (including Bose, Fermi and classical ones corresponding to $q=\pm1$ and 0, respectively) in thermal equilibrium, we compute the asymptotic expansion with respect to the natural cut-off given by (a function of) the inverse temperature. We treat both relevant situations relative to massless and non relativistic massive particles, where the natural cut-off is $1/\b=k_{\rm B}T$ and $1/\sqrt{\b}$, respectively. We show that the massless situation enjoys less regularity properties than the massive one. We also treat in some detail the relativistic massive case for which the natural cut-off is again $1/\b$. We then consider the passage to the continuum describing infinitely extended open systems in thermal equilibrium, by also discussing the appearance of condensation phenomena occurring for Bose-like q-particles, $q\in(0,1]$. The situations relative to the finite volume (discrete spectrum) and the infinite volume (continuous spectrum) are studied and compared.
The more singular situation corresponding to the massive case is handled by using the theory of distributions associated to a very particular class of test-functions, the last having connections with the Riemann $\z$-function.

The present talk is based on: F. Ciolli and F. Fidaleo Spectral actions for q-particles and their asymptotics, J. Phys. A (Math. Theor.) 55 (2022), 424001 (19 pp).

Johannes Große (FAU Erlangen-Nürnberg**)**

KMS states on Z_2-crossed products and twisted KMS functionals

Abstract: We study the extensions of a KMS state on a graded unital C*-dynamical system to its crossed product by Z_2. In this talk, we show that dominated twisted KMS functionals describe the different extensions of the KMS state to the Z_2-crossed product. These functionals can also be characterized by the twisted center of the von Neumann algebra generated by the GNS representation corresponding to the KMS state. This allows one to give an upper bound on the number of extremal extensions of the KMS state.

As a particular class of examples, KMS states on Z_2-crossed products of CAR algebras with dynamics and grading given by Bogoliubov automorphisms are analyzed in detail. In this case, one or two extremal KMS states are found depending on a Gibbs type condition involving the odd part of the absolute value of the Hamiltonian.

This talk is based on the preprint arXiv:2402.15574 and is a joint work with Ricardo Correa da Silva and Gandalf Lechner.

Larissa Kroell (University of Waterloo)

The intersection property of partial reduced crossed products

Abstract: Given a C*-dynamical system, a fruitful avenue to study its properties has been to study the dynamics on its injective envelope. In particular, Kennedy and Schafhauser (2019) give a characterization of the ideal intersection property of a C*-dynamical system in terms of proper outerness of the group action on its G-injective envelope under some assumptions. In this talk, we generalize these techniques to partial C*-dynamical systems and show that if the system is properly outer, it exhibits the intersection property. In order to do so, we introduce the category of generalized partial actions and show that the intersection property can be characterized using equivariant pseudo-expectations similar to the case of ordinary C*-dynamical systems. This is joint work with Matthew Kennedy and Camila Sehnem.

Sarah Reznikoff (Virginia Tech)

TBA

Abstract

Dan Ursu (Westfälische Wilhelms-Universität Münster)

Simplicity of crossed products by FC-hypercentral groups

Abstract: Results from a few years ago of Kennedy and Schafhauser characterize simplicity of reduced crossed products AxG, where A is a unital C*-algebra and G is a discrete group, under an assumption which they call vanishing obstruction. However, this is a strong condition that often fails, even in cases of A being finite-dimensional and G being finite. In joint work with Shirly Geffen, we find the correct two-way characterization of when the crossed product is simple, in the case of G being an FC-hypercentral group. This is a large class of amenable groups that, in the finitely-generated setting, is known to coincide with the set of groups which have polynomial growth. With some additional effort, we can characterize the intersection property for AxG in the non-minimal setting, for the slightly less general class of FC-groups. Finally, for minimal actions of arbitrary discrete groups on unital C*-algebras, we are able to generalize a result by Hamana for finite groups, and characterize when the crossed product AxG is prime. All of our characterizations are initially given in terms of the dynamics of G on I(A), the injective envelope of A. This gives the most elegant characterization from a theory perspective, but I(A) is in general a very mysterious object that is hard to explicitly describe. If A is separable, our characterizations are shown to be equivalent to an intrinsic condition on the dynamics of G on A itself.

Qin Wang (East China Normal University)

Ideal structure of uniform Roe algebras

Abstract: The uniform Roe algebra of a discrete metric space is a C^*-algebra that analytically encodes the coarse geometry of the underlying space. It is known that counter-examples to the coarse Baum-Connes conjecture can be constructed by using expander graphs, the crucial point of which lies in the fact that the uniform Roe algebra of the metric space of a sequence of expander graphs contains a noncompact projection P, called the ghost projection, which is locally invisible at infinity. In terms of ideal structure, accordingly, the finite propagation operators in the principal ideal generated by P, are not dense in the ideal. We will first focus on the class of ideals of a uniform Roe algebra in which finite propagation operators are dense, and show that these ideals can be described geometrically in terms of coarse structure and invariant open subsets of the unit space of the Skandalis-Tu-Yu groupoid. We show that if the metric space has Yu's property A, then all ideals are geometric. We introduce a notion of ghostly ideal and partial property A to investigate the ideal structure of the uniform Roe algebra for a general metric space beyond the scope of Yu's property A. This talk is based on joint works with Xiaoman Chen and Jiawen Zhang.